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Improving mathematical learning in Scotland's *Curriculum for Excellence* through problem posing: an integrative review

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The purpose of this paper is to explore the importance of problem posing in learning mathematics at the compulsory education level. Despite acknowledging that children have a natural disposition to pose questions, no curricular provision currently exists for the conceptualisation and operationalisation of mathematical problem posing within Scotland's Curriculum for Excellence. In order to provide evidence to support any curricular change, integrative systematic review and narrative synthesis of quantitative and qualitative studies was conducted. Results suggest that problem posing can offer an array of valuable didactic benefits for pupils such as deeper conceptual knowledge, enhanced problem-solving skills and an increase in the enjoyment of mathematics. Evidence from the qualitative synthesis provides some tentative guidance on considerations regarding the integration of problem posing to the curriculum. This study argues that in order to improve future learning experiences, mathematical problem posing should be embedded in all Scottish classrooms. Furthermore, problem posing is determined to be effective in the pedagogical development of prospective primary and secondary mathematics practitioners.

Keywords: practitioner researcher; mathematical problem posing; curricular policy; Scotland; teacher education; integrative review

Introduction

In this paper, we argue that curriculum architects have insufficiently recognised the growing body of work on problem posing during the development of CfE and its reconceptualisation of the learning and teaching of mathematics. Through the process of practitioner research, it draws on findings from the first author (McDonald, 2017). Teaching as a research-informed profession has become an important mantra within Scottish education and has sought to position teachers as prime agents of change (e.g. Donaldson, 2011; GTCS, 2012). However, Muschamp (2013) points out that although the professional guidelines do not require a teacher to be research active, they nonetheless infer a tacit ability to interpret and evaluate findings. In other words, being in possession of research skills. This paper is, therefore, the representative of the duty teachers to act purposefully by shaping curricula policy through

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critical inquiry (e.g. Priestley *et al.*, 2015) and to be co-producers of knowledge (Maclellan, 2014).

Mathematics is of central importance to daily living. Entrenched societal attitudes towards mathematics are barriers to educational and economic success (Scottish Government, 2016). Recent trends in international mathematical performances show that Scotland has a growing proportion of low achievers and a shrinking proportion of high achievers (e.g. OECD, 2015). A perpetual challenge for educators is to connect mathematics to real life. Furthermore, prospective primary teachers often report a lack of self-efficacy in mathematics which can impact adversely on classroom experiences (e.g. Henderson, 2012). There is an urgent requirement to address confidence and fluency levels in mathematics education, including the need to raise attainment and achievement across learning (Scottish Government, 2016). Problem posing is considered by many stakeholders as a pedagogy that can augment the learning and teaching of mathematics (e.g. Cai *et al.*, 2015; Singer *et al.*, 2015).

Whilst it has been acknowledged that problem posing is an inseparable component of problem solving (e.g. Ellerton, 2013), misconceptions exist regarding the structure of a mathematical problem. For example, in a study of 478 Scottish primary and mathematics teachers' beliefs, McDonald (2017) found that almost half of participants believed that a mathematical problem is classified by the union of words and a routine algorithmic task. Similarly, Xonofontos and Andrews (2012, 2014) reported that mathematical beliefs of primary teachers were misplaced about the conceptualisation of a problem. Within the literature, a mathematical problem is an unfamiliar task that requires a level of cognitive challenge to stimulate curiosity and critical thinking (Schoenfeld, 1985; Mason *et al.*, 2010). Instrumental to effective teaching of problem posing is an explicit understanding of the nature of a mathematical problem. Authentic mathematical problems can be identified by several characteristics promoting higher-order thinking, engaging the solver and embodying important mathematics (Kilpatrick, 1987). The following real-life event can be solved in alternative ways making it appealing to a range of age groups and abilities:

How many different ways can a group of four children line up together outside a classroom?

Mathematical problem posing

Kilpatrick (1987) highlights three categories of problems: Well-structured problems are overtly formulated, can be solved by the application of a known algorithm and the solution can be tested against criteria; structured problems are similar but require the solver to contribute to the solution; ill-structured problems lack a clear formulation, a procedure that will guarantee a solution and criteria for determining when a solution has been achieved.

Problem posing encompasses the generation of new problems and the reformulation of given problems (e.g. Silver & Cai, 1996; English, 2004; Whitin, 2006). New problems can emerge before or after the problem-solving process and reformulation follows when the original problem is transformed into a different version (Silver, 1994). Problem posing can also be based on ill-structured problems (Pirie, 2002).

However, such an approach does not provide clarity on the extent of previous knowledge required; ill-structured problems can be situated in engaging, realistic contexts but often require the application of multiple subject domains outside mathematics (Toy, 2007).

Stoyanova and Ellerton (1996, p. 518) link constructivism to problem posing and suggest it is 'the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems'. The researchers propose a framework grounded on three types of problem-posing situations: In free situations, problems are derived from real life without restrictions; semi-structured demands creative imagination as an open situation is explored using knowledge from previous mathematical experiences; structured activities are centred on a specific problem that requires completion or reformulation. Although such conditions can spawn a diversity of topics, a specific mathematical theme can similarly be employed. For instance, Canadas *et al.* (2018) present situations using algebraic statements to extend mathematical thinking as opposed to performing meaningless operations on equations. Table 1 displays posed problems from the previous research.

Problem posing can help pupils prepare for future workplace challenges through enhanced creativity skills (Brown & Walter, 2005). However, despite problem solving being present throughout schooling, most problems come from textbooks. The main disadvantage of textbooks is that they do not always relate to the needs and interests of learners; dependency is commonly associated with a teachers' lack of pedagogical content knowledge (Gracin & Matic, 2016). Brown and Walter (2005) encourage a move to constructing and designing problems within the classroom.

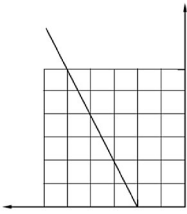
Problem posing is not an original concept. Einstein (Einstein & Infeld, 1938) championed the notion when he suggested:

The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill. To raise new questions, a new possibility, to regard old problems from a new angle, requires creative imagination and marks real advances in sciences.
(p. 92)

Historically, this view has been shared by others who have placed greater emphasis on posing meaningful questions than on solving them. For example, Socrates (470–399BC) 'established an efficient method of learning through a continuous dialogue based on posing and answering questions to stimulate critical thinking and illuminate ideas' (Singer *et al.*, 2013, p. 2).

More recently, although problem posing has gained increasing awareness amongst educationalists, a lack of consensus regarding classroom approaches could limit its future development. For example, problem posing has been marginalised by the research community (English, 1998; Crespo, 2003; Leung, 2013), despite the assertion that it is an important element in developing critical thinking skills (e.g. Ellerton, 1986; Silver, 1994; Singer *et al.*, 2013). However, researchers argue that it should be granted comparable status as problem solving (Silver *et al.*, 1990; Pirie, 2002; Stoyanova, 2003; Silver & Cai, 2005). This would allow a research focus on evidenced-based strategies for school integration and within initial teacher education (ITE) (e.g. Singer *et al.*, 2015).

Table 1. Examples of posed problems from previous research

Task	Source	Participants	Sample
What would you say would be a good story problem or model for $1\frac{3}{4} \div \frac{1}{2}$?	Ma (1999)	In-service primary teachers	Yesterday I rode a bicycle from town A to town B. I spent $1\frac{3}{4}$ hour for $\frac{1}{2}$ of my journey, how much time did I take for the whole journey?
Pose a similar problem to the following one: ‘The side lengths of a triangular-shaped field, whose perimeter is 512 meters, are proportional with the numbers of 4, 5 and 8. How long is the longest side of this field?’	Şengül and Katranci (2014)	Prospective primary mathematics teachers	The interior angles of a triangle 2, 3 and 4 are proportional to the number of regions. How many degrees is the smallest angle of this triangle?
Write three different questions that can be answered from the information below: Jerome, Elliot and Arturo took driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove as many miles as Jerome. Jerome drove 50 miles	Silver and Cai (2009)	Primary pupils	How many more miles did Elliot drive than Jerome? How many miles did the boys drive altogether?
Make up as many problems as you can using the following calculation: $3 \times 25 + 15 \div 5 - 4$	Stoyanova (1996)	Secondary pupils	Did Arturo drive a longer time than Jerome and Elliot drove all together in the regular way? Around which two digits could you place brackets so that the answer is 80?
<div>  </div>	Cai <i>et al.</i> (2013)	Secondary pupils	The cost of renting a bike is a \$2 payment plus \$0.50 for each day you keep it.

Write a real-life situation that could be represented by this graph. Be specific

The role of problem posing in mathematics

Theorists suggest that problem posing has a central role in the learning of mathematics. Pupils cannot fully experience mathematics unless they solve problems created by themselves (Polya, 1957). Therefore, all pupils should experience creating their own mathematical problems (Kilpatrick, 1987). This activity is accessible to all learners regardless of age or ability. Indeed Lowrie (2002) found that children as young as five were able to pose problems.

Problem posing may help teachers to assess learners' conceptual understanding, problem solving and creativity (e.g. Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996; Silver, 1997; English, 1997a, 1997b; Cai & Hwang, 2002; Lowrie, 2002; Van Harpen & Sriraman, 2013). For example, learners show differential retention for specific aspects of problems when asked to engage in problem-solving activities. Krutetskii (1976) studied problem-solving abilities of highly able pupils. He noted differences in the way in which they remembered the problems that they had solved. Specifically, highly able pupils remembered generalised structural components of problems they had solved up to 3 months after they were introduced. Concrete (surface) information on the problem, together with superfluous information, was retained initially but not to the same extent as the structural information. However, pupils who were not categorised as highly able were more likely to remember the surface information about the problem. For example, a highly able learner may recall: 'I did a problem on different combinations of the parts of a whole—about a fish whose tail and head weigh so much...' and a struggling pupil may recall they completed a problem 'something about a fish weighing 2 poods [sic]' (Krutetskii, 1976, p. 299). Asking pupils to pose problems based on previously solved problems could, therefore, allow teachers to evaluate conceptual understanding.

In their study of primary pupils, English and Watson (2015) investigated the impact of problem posing on developing statistical literacy. They found that participants worked creatively and critically on tasks and that problem posing has the power to develop thinking and improve confidence. Cai *et al.* (2013) studied the long term effect on the learning of secondary pupils. Using a system of linear equations, the researchers found a strong relationship between the ability to solve a problem and the capacity to pose problems.

Teachers can challenge learners to think deeply about what they are doing rather than mechanically respond to a set of questions with a prepared technique or algorithm. Likewise, problem posing can be empowering as it encourages pupils to construct knowledge (e.g. Ernest, 1991; English, 1997a) and decide on questions to be solved. This challenges the assumptions that there is only one method to solve a problem and that all problems have one correct answer (Fox & Surtees, 2010). Problem posing can create a dynamic learning environment where children are inspired to take risks and are less afraid to make mistakes. Whitin (2004, p. 129) asserts that it can enhance the atmosphere of every classroom and portrays it as 'a strategy that builds a spirit of intellectual excitement and adventure by legitimizing asking questions and freeing learners from the one-answer syndrome', thereby encouraging them to explore numerous scenarios. Crucially, when learners generate their own problems, they probe and crystallise their mathematical knowledge more deeply than

when handed a litany of ready-made facts (Watson & Mason, 2005). In the same vein, it has been argued that problem posing can help stimulate diverse and flexible reasoning (e.g. Silver, 1994; Leung, 2013; Kwek, 2015), foster creativity (e.g. Silver *et al.*, 1990; Silver, 1997; Van Harpen & Sriraman, 2013), eliminate textbook dependency (e.g. Brown & Walter, 2005) and support the promotion of independent learning and critical thinking skills (e.g. Kilpatrick, 1987; Silver, 1994; Silver & Cai, 1996; Brown & Walter, 2005; Mamona-Downs & Downs, 2005), which are the cornerstones of CfE.

Unsurprisingly, problem posing has featured within worldwide curricula reforms. Curriculum reform is a powerful driver for implementing change within educational systems (Cai & Howson, 2013). In America, the NCTM (1989, p. 138) endorsed problem posing by promoting that ‘students in grade 9-12 should also have some experience recognising and formulating their own problems, an activity that is at the heart of doing mathematics’. During a later reform, the NCTM (2000) declared the function of the teachers is to orchestrate such opportunities for all pupils. Stoyanova and Ellerton (1996) reported that the Australian Education Council (1991) offered an endorsement for the use of open-ended problems. China has bestowed attention to problem posing alongside problem solving (e.g. Cai & Nie, 2007). Within Singapore, pupils are encouraged to extend and generate problems (e.g. Ministry of Education, 2007).

Nevertheless, curricula reform is not exempt from conflict. For example, the inclusion of problem posing has promulgated tension in Taiwan. Teachers are facing unprecedented challenges to change their pedagogy to assimilate problem solving and problem posing (Leung, 2013). This highlights the need for training and resources to counter inexperience and implementation difficulties (e.g. Leung, 1994). Italy (e.g. Bonotto & Del Santo, 2015) and Turkey (e.g. Kılıç, 2013) have introduced reforms to embed problem posing across education levels. However, research suggests that the ability to pose suitable tasks is correlated with problem-solving competence in practitioners (e.g. Crespo, 2003; Koichu & Kontorovich, 2013). Moreover, workload pressures inhibit teachers from producing resources. Though, problems can be generated from other networks. In a study of 70 Portuguese prospective primary teachers, Barbosa and Vale (2016) explored authentic contexts outside the classroom contributing to the posing of mathematical problems such as monuments, windows and gardens. Building on the work of Silver (1997) and Stoyanova (1998), the researchers analysed personal interpretations and formulations of real situations inspired by the local environment (in one case, iron railings were linked to geometric shapes). They found that participants displayed a more positive attitude towards learning and teaching of mathematics by acquiring a broader view of the connections between the natural worlds. This change may enrich settings for pupils to discover and construct knowledge.

Location of mathematical problem posing within CfE

Whilst mathematical problem posing is not theoretically conceptualised within CfE, there is the potential to build on initial curricula development guidelines. For example, the Scottish Executive (2007) stated that:

Children have a natural disposition to wonder, to be curious, to pose questions, to experiment, to suggest, to invent and to explain. Staff have an essential role in extending and developing this.
(p. 13)

In a later publication, the Scottish Government (2010, p. 1) describes the function of teachers in supporting active learning 'through engaging the learner in dialogue, asking questions, posing problems'. Taken together, these guidelines support the notion that problem posing is an existing goal and a critical aspect of the nurturing work of teachers. However, it raises a key issue of equality. Enshrined in legislation is the right for every child and young person to expect appropriate assistance to allow them to reach their full potential. Moreover, it is thought that the teaching of mathematics is a social justice issue (e.g. Kaur, 2012). Thus, it seems plausible that a pupil may be disadvantaged if their educational experience does not make provision for problem posing.

Potential limitations to mathematical problem posing in the classroom

Since problem-posing duties are fostered by pedagogical actions, it is vital that teachers are trained accordingly (Lowrie, 2002; Leung, 2016). Koichu *et al.* (2013) found that mathematics teachers require help to understand that problem posing is a fundamental part of education. Ellerton (2013) warns that any adjustment to professional practice must be accompanied by a focus on problem posing during ITE. However, studies have exposed a shortcoming with prospective teachers' problem-posing skills (e.g. Crespo, 2003; Chapman, 2012). Within Scotland, another impediment might be how to find space within a saturated curriculum. Likewise, there is the interrelated matter of assessment. McDonald (2017) argues that the operationalisation of both problem solving and problem posing is circumscribed in practice without a corresponding assessment framework. It is unknown if such a systemic change will be embraced by teachers.

Furthermore, evidence suggests that non-mathematical barriers may exist to learners accessing problem posing and problem solving due to their linguistic nature. It can be difficult for learners to access problem solving to the linguistic and semantic structure of word problems (e.g. Boonen *et al.*, 2013). Problem posing also presents similar challenges. For example, Cheng (2013) found that inadequate vocabulary understanding contributed to primary pupils' inability to pose valid fraction problems. Regardless, research demonstrates that engaging in tasks that require pupils to generate word problems, reading comprehension can be enhanced (e.g. Rosenshine *et al.*, 1996; Yang & Lin, 2012). It is, therefore, an important consideration for educators to ensure that learners are supported effectively through problem posing, both to be able to access the mathematical content through words, but to develop their reading skills and comprehension. Despite these potential drawbacks, evidence suggests that the advantages would outweigh the challenges of implementation.

There is hence the extensive theoretical justification for incorporating problem posing within the CfE. Furthermore, problem posing as a concept can be operationalised to allow classroom enactment. Whilst numerous countries have successfully implemented problem posing, it is still not part of the Scottish educational landscape; that is, despite the availability of empirical evidence verifying the benefits for pupils and teachers. So far, two systematic literature reviews have been published. In their

meta-analysis, Rosli *et al.* (2014) reported rewards for the learning and teaching of mathematics from thirteen experimental studies published between 1989 and 2011. Consequently, there has been a growth in studies in this area. More recently, Zuya (2017) revealed benefits to learners of mathematics in his review of 16 experimental studies published up to 2016. However, Zuya (2017) did not clarify his search strategy nor did he specify methodological quality criteria. Furthermore, Zuya's study did not purposely consider potential benefits for teachers.

The requirement to consider teachers is grounded on a growing body of research which has acknowledged problem posing as a valuable tool in developing mathematics teaching at all levels (e.g. Pittalis *et al.*, 2004; Cai *et al.*, 2015; Ellerton, 2015). Ticha and Hospesova (2013) found that prospective primary teachers acquired a deeper conceptual understanding of fractions. Likewise, in their study of prospective mathematics teachers, Lavy and Shriki (2010) discovered an increase in geometric knowledge and curiosity and enthusiasm towards mathematics. Hospesova and Ticha (2015) found that problem posing within ITE is an effective method of enhancing didactic competence. Equally, Crespo (2015) maintains that without problem-posing training, prospective teachers will enter the profession with limited vision and strategies. Consequently, there is a need for a new systematic review which is both methodologically rigorous and considers the benefits of problem posing for pupils, teachers and prospective teachers.

Research question

To what extent should problem posing be embedded within the mathematical framework of Curriculum for Excellence (CfE)?

Secondary research questions

1. What are the benefits for learners of using mathematical problem posing in the curriculum?
2. What are the benefits for ITE students using mathematics problem posing?

Methods

Integrative systematic review and narrative synthesis was conducted in order to allow a robust and reproducible approach to the synthesis of existing research. A preliminary search identified a range of evidence on the benefits of problem posing in relation to the review questions. This provided guidance for the support of a narrative synthesis of findings from heterogeneous studies (Popay *et al.*, 2006). The Cochrane and Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA Moher *et al.*, 2009) guidelines were followed to produce and report a systematic and rigorous review of both quantitative and qualitative literature.

Eligibility criteria

Inclusion criteria were as follows; studies which contained an intervention, or used problem posing in mathematics (a distinction is made between such studies and

those which merely ask participants to engage in problem-posing activities as part of the study); participants were either pupils (at any stage) or ITE students (primary or mathematics); quantitative studies of quasi-experimental nature should have appropriate statistical analysis; both qualitative and quantitative studies should have sufficient methodological information supplied. Exclusion criteria were: Studies published in a language other than English; studies published prior to 1996.

Information sources

The following electronic databases were searched for the published literature: PsychInfo, ERIC, JSTOR and ScienceDirect. Reference lists of relevant articles or reviews were also examined. Unpublished literature was searched in the ProQuest Dissertations & Theses Global database.

Search and study selection

Key search terms were employed in each database using the review questions as a guide. Search terms were (students OR teachers OR pupils) AND math* AND 'problem posing'. The search was completed during August 2019 and returned 1317 articles, reducing to 1193 after duplications were removed. A summary of the selection process is provided in the PRISMA flow diagram (Figure 1). All articles (100%) were screened by both authors independently by referring to the title and abstract to ascertain the likelihood of meeting eligibility criteria. Inter-rater reliability prior to discussion and consensus was measured by randomly checking 25% of the articles. Cohen's Kappa coefficient was calculated to be 0.934 suggesting almost perfect agreement (Landis & Koch, 1977). All potential remaining titles (219) were divided between the two authors and were read fully to establish if they were eligible.

Data extraction and analysis

Data extraction tools were developed *a priori* for both quantitative data (Table 2) and qualitative data (Table 3). These were piloted on randomly selected included studies (four quantitative and two qualitative). Data on effect sizes were extracted for quantitative studies, where present. Where studies did not report relevant effect sizes, the first author calculated these if appropriate statistics were available. Across studies, some necessary statistical information was unavailable. Due to the heterogeneity of outcome measures and study populations in the quantitative and qualitative studies, a meta-analysis was not deemed appropriate. Quantitative results were synthesised narratively.

Data analysis

The qualitative data were synthesised thematically in order to understand the benefits of problem posing, as well as factors that might contribute to, or hinder, successful problem posing. This was in order to make practical recommendations for the

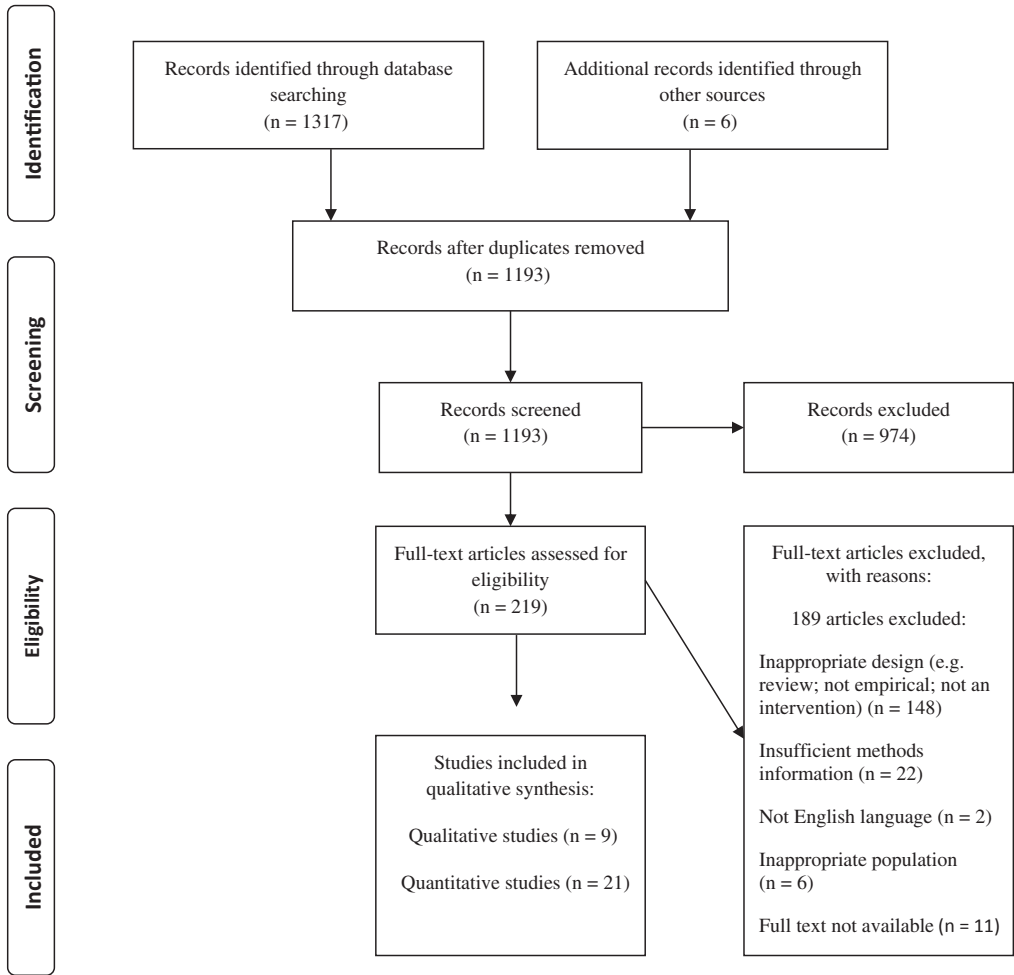


Figure 1. PRISMA flow diagram (Moher *et al.*, 2009)

implementation of problem posing to the curriculum (Popay *et al.*, 2006). Qualitative data were analysed using thematic analysis which allows the identification and analysis of themes (Braun & Clarke, 2006). The six-step process described by Braun and Clarke (2006) was utilised and details are shown in Table 4. Nowell *et al.* (2017) note that trustworthiness in the thematic analysis is derived from describing the process of data analysis clearly. The qualitative papers were first read in-depth, taking notes on relevant information related to the review questions. This would allow further category identification. Theme identification was initially conducted in relation to the review questions. Themes were then drawn from the initial categories with particular reference to the aims of the paper to consider if problem posing should be implemented within CfE. Specifically, themes relating to the implementation of problem posing were highlighted, in line with Popay *et al.*'s (2006) guidance on narrative reviews. Results of studies were used in the data analysis, with the discussion and authors' conclusions being considered in relation to their findings. The process was

Table 2. Study characteristics quantitative data

Author and year	Country	Participants	Intervention	Outcomes measured	Study design	Results	Effect size(s)
Abu-Elwan (2002)	Oman	Secondary 50 ITE mathematics students (25 control, 25 experimental)	Seven weeks problem-posing reformulation tasks developed by the researcher	Mathematical problem solving-posing achievement test designed by the researcher	Quasi-experimental	Significant improvement in (a) problem-solving performance (b) problem-posing performance (c) problem (solving-posing) for the experimental group compared with the control group	(a) 0.59* (b) 0.83* (c) 0.94*
Akay and Boz (2009)	Turkey	Primary 79 ITE students (38 control, 41 experimental)	Eight weeks problem-posing activities developed by the researcher based on 'What-if-Not' strategies (Brown & Walter, 1983) and 'Structured, Semi-Structured & Free situations' (Stoyanova & Ellerton, 1996, 1998)	Calculus performance test designed by the researchers	Quasi-experimental	Significant improvement in mathematics performance for the experimental group compared with the control group	0.46**
Akay and Boz (2010)	Turkey	Primary 82 ITE students (42 control, 40 experimental)	Ten weeks based on problem-posing strategies related to integration	Mathematics attitude scale (Askar, 1986) Mathematics self-efficacy beliefs scale (Umay, 2001)	Quasi-experimental Groups selected randomly	(a) The attitude towards mathematics was significantly more positive for the experimental group than for the control group (b) Mathematics self-efficacy beliefs were significantly stronger for the experimental group than for the control group	(a) 0.53* (b) 0.54*
Barlow and Gates (2006)	USA	Primary 61 in-service teachers	One year professional learning programme incorporating problem posing into lessons	Beliefs questionnaire based on Knight (1991), Zambo (1994), Riley (1999) and researchers	Pre and post questionnaire	Positive change in beliefs about mathematics and mathematics teaching	0.50*
Chen <i>et al.</i> (2015)	China	Primary 69 pupils 11-12 years (36 control, 33 experimental)	Eleven week intervention based on learning tasks, instructional techniques & socio-mathematical norms (Rudnitsky <i>et al.</i> , 1995; English, 1997a, 1997b, 1998; Winograd, 1997; Verschaffel <i>et al.</i> , 2000)	Problem-Posing Test (arithmetic, geometry & statistics) Problem-Solving Test (arithmetic, geometry & statistics) Problem-Posing Questionnaire Problem-Solving Questionnaire Standard Achievement Test (Shenyang Municipal Educational Committee)	Design experiment	(a) The originality of the problems posed by the experimental group was significantly better than for the control group. Further evidence included (b) significantly better problem-solving performances and (c) more positive beliefs and (d) attitudes towards problem posing and problem solving	(a) 0.11* (b) 0.57* (c) 1.02* (d) 1.27*

(Continues)

Table 2. (Continued)

Author and year	Country	Participants	Intervention	Outcomes measured	Study design	Results	Effect size(s)
Demir (2005)	Turkey	Secondary 82 pupils 15-16 years (55 control, 27 experimental)	Six-week intervention using problem-posing activities based on Stoyanova and Ellerton (1996)	Probability Achievement Test (Researcher) Probability Attitude Scale (Bulut, 1994) Mathematics Attitude Scale (Askar, 1986)	Experimental Random selection (unspecified technique)	(a) Significant improvement in probability attainment for the experimental group compared with the control group. Further evidence noted improved attitude towards (b) probability and (c) mathematics	(a) 0.25** (b) 0.25** (c) 0.20**
Dickerson (1999)	USA	Secondary 210 pupils 12-13 years (52 control, 158 experimental)	Two-year problem-posing intervention based on 'structured', 'acting-out', 'open-ended' & 'what-if-not' strategies	Problem-Solving Achievement Total Mathematics Battery	Quasi-experimental	Significant improvement for the experimental group compared with the control group	0.29**
English (1998)	Australia	Primary 54 pupils 7-8 years (27 control, 27 experimental)	Two-month problem-posing intervention based on addition and subtraction contexts including activities dealing with novel, non-operational problem situations	Diversity of problem creation Ability to pose problems in different contexts	Quasi-experimental	The experimental group demonstrated a significant improvement in the ability to generate mathematical problems compared with the control group. Increase in multi-step problems	
Fetterly (2010)	USA	Primary 32 ITE students (16 control, 16 experimental)	Fifteen-week problem-posing intervention programme based on multiple perspectives, open-ended problems, sample solutions and alternative problems	The Mathematics Belief Questionnaire (Collier, 1972) The Abbreviated Math Anxiety Scale (Hopko <i>et al.</i> , 2003) Creativity Ability in Mathematics (Balka, 1974) General Assessment Criteria (Silver & Cai, 2005)	Quasi-experimental Convenience sampling	(a) Problem posing can foster and sustain mathematical creativity. Problem posing had a significant positive impact on (b) mathematical beliefs and reducing mathematical anxiety (c) for the experimental group compared with the control group	(a) 3.53* (b) 8.78* (c) 4.44*
Grundmeier (2003)	USA	Primary & Secondary (Up to age 14) 19 ITE students	Fifteen weeks incorporation of problem-posing activities into a mathematics content course. Participants solved problems using Polya (1957) heuristic and then posed related problems	Problem reformulation, problem generation and beliefs	Exploratory	Positive change in problem-posing generation and re-formulation ability (Leung & Silver, 1997) Positive changes in beliefs about Mathematics Positive changes in beliefs about teaching and learning of mathematics Positive change in beliefs about the relationship between problem posing and mathematics teaching and learning	

(Continues)

Table 2. (Continued)

Author and year	Country	Participants	Intervention	Outcomes measured	Study design	Results	Effect size(s)
Guvercin <i>et al.</i> (2014)	Kazakhstan	Secondary 60 pupils 15-16 years (30 control, 30 experimental)	Seven weeks using problem-posing theory with triangular problems	Mathematical Achievement Retention of triangular problems	Mixed methods experimental Achievement test Interview via video recording	Significant increase in the mathematical academic achievement of the experimental group compared with the control group. Further evidence included a significant visual effect on retention and a positive attitude towards mathematics	0.23**
Guvercin and Verbovsky (2014)	Kazakhstan	Secondary 54 pupils 14-15 years	Seven weeks using problem-posing activities	Mathematics Achievement Test Mathematics Attitude Scale (Fennema & Sherman, 1986)	Mixed methods experimental	Significant increase in the mathematical academic achievement of the experimental group compared with the control group. Further evidence included positive attitude towards mathematics and increased levels of motivation and cognitive thinking	0.25**
Haghverdi and Gholami (2015)	Iran	Secondary 29 pupils Unspecified	Six month intervention using problem-posing activities based on the 'What if Not?' strategy	Generation of computational geometric problems Generation of proof geometric problems	Pre and posttest	Significant increase in the number of relevant problems posed by the experimental group compared with the control group. Problem posing strengthened the understanding of connections between geometric concepts	0.01**
Kesan <i>et al.</i> (2010)	Kazakhstan	Secondary 40 pupils 14-15 years	Eight week intervention using problem-posing activities (Stoyanova, 2000) Activities based on mathematics, physics and statistics	Mathematical Problem-Solving Test	Quasi-experimental	Enhanced motivation and improved flexible thinking of the experimental group compared with the control group. Furthermore, greater classroom interaction resulting in increased mathematical performance	0.39**
Kopparla <i>et al.</i> (2018)	USA	Primary 45 pupils 7-11 years	Seven weeks based on problem-posing activities	A problem-solving and problem-posing quiz	Quasi-experimental	Significant improvement in (a) problem-solving skills and (b) problem-posing skills for the experimental group compared with the control group	(a) 0.59 [†] (b) 0.44 [†]

(Continues)

Table 2. (Continued)

Author and year	Country	Participants	Intervention	Outcomes measured	Study design	Results	Effect size(s)
Mahendra <i>et al.</i> (2017)	Indonesia	Secondary 63 pupils 11-12 years (32 control, 31 experimental)	Two months based on problem posing and problem solving	Conceptual Comprehension Test on Geometry Mathematical Adaptive Reasoning in Geometry	Quasi-experimental (stratified cluster sampling)	Significant improvement in conceptual understanding of geometry for the experimental group compared with the control group. Improved adaptive reasoning	0.51*
Ozdemir and Sahal (2018)	Turkey	Primary 69 pupils 11-12 years (35 control, 34 experimental)	Five weeks using lesson plans focussing on problem posing. Students asked to create new problems based on a given example. Then asked to pose problems based on a story or piece of information using real data	The Mathematics Attitude Scale (Erkin & Nazlicicek, 2002) Integers subject Achievement Test Problem-Posing Evaluation Rubric (Katranci, 2014)	Quasi-experimental	(a) The significant improvement between the Mathematics Attitude Scale for the experimental group compared with the control group (b) Significant improvement between the Achievement Test in Integers for the experimental group compared with the control group	(a) 0.19 ^{††} (b) 0.52*
Priest (2009)	Australia	Primary 31 pupils 11-12 years (16 control, 15 experimental)	Seven week problem-posing intervention consistent with a critical theorist approach	Students' engagement Problem-solving competence	Mixed methods experimental	The intervention facilitated the re-engagement of pupils from the experimental group compared with the control group. Further evidence included improved problem-solving competence and the facilitation of developmental learning	0.78**
Toluk-Ucar (2009)	Turkey	Primary 95 ITE students (50 control, 45 experimental)	One term problem-posing intervention on discussions of the appropriateness of generated word problems. The focus was on the justifications of the posed problem using different modes of representations	A fraction test An open-ended question about how individuals perceive their fraction knowledge Weekly mathematics journals	Quasi-experimental	The experimental group demonstrated a positive impact on the understanding of fractions and on views about what it means to know mathematics compared with the control group	0.79**
Walkington (2017)	USA	Secondary 171 pupils 13-14 years (77 experimental, 94 control)	Four day intervention employed personalised learning on posing and solving algebraic problems based on students shared out of school interests	Questionnaire (Bandura, 2006; Linnerbrink-Garcia <i>et al.</i> , 2010) Measure of situational interest Measure of self-efficacy Algebraic performance	Design research	Significant improvement in algebra performances for the experimental group compared with the control group	0.35*

(Continues)

Table 2. (Continued)

Author and year	Country	Participants	Intervention	Outcomes measured	Study design	Results	Effect size(s)
Xia <i>et al.</i> (2008)	China	Secondary 540 pupils 12-15 years	Two-year problem-posing intervention using teaching model of 'Situating Creation and Problem-based Instruction' (SCPBI) i.e. the teaching process of creating situations posing problems and solving problems applying mathematics	Questionnaire (Lu & Wong, 2006)	Quasi-experimental	Significant effect on improving (a) interest in mathematics and (b) the ability to learn mathematics was found for the experimental group compared with the control group	(a) 0.16** (b) 0.04**

Effect size intervention guidelines (Cohen *et al.*, 2017).

*Cohen's *d* where: 0.2 = Small effect, 0.5 = Medium effect and 0.8 = Large effect (Cohen, 1988). **Eta squared where: 0.01 = Small effect, 0.06 = Medium effect and 0.14 = Large effect (Cohen, 1988). †Hedge's *g* where: 0.2 = Small effect, 0.5 = Medium effect and 0.8 = Large effect (Cohen, 1988). ††Nonparametric test data where: 0.1 = Small effect, 0.3 = Medium effect and 0.5 = Large effect (Cohen, 1988).

Table 3. Study characteristics qualitative data

Authors and year	Country	Participants and context	Aims	Design	Analysis	Main Findings
Ellerton (2013)	USA	Primary/Secondary middle school 154 ITE students	To investigate the viability of integrating problem-posing activities into the curriculum in parallel with problem-solving activities	Exploratory study	Descriptive analysis of quantitative and qualitative data	Preservice teachers were able to pose problems within the context of problem-solving work, although often they included imperfections in wording or logic
English (1997b)	Australia	Primary 27 pupils	to explore the extent to which children's number sense and novel problem-solving skills govern their problem-posing abilities in routine and non-routine situations	Exploratory study	Descriptive analysis of interview data	Pupils displayed difficulties in recognition and utilisation of problem structures and diverse mathematical thinking. These were addressed to different extents through the implementation of problem posing in the mathematics lessons. Pupils also initially reported a limited range of problems that they would like to pose. However, this increased throughout the programme
Grundmeier (2015)	USA	Primary/Secondary middle school 9 ITE students	To explore the impact of the integration of problem posing during the mathematics content course	Exploratory study	Descriptive analysis of qualitative and quantitative data	Participants developed more sophisticated problem reformulation techniques. They also developed more efficient ways of posing problems. Their ability to pose multi-step problems also improved. Participants developed beliefs that the use of problem posing in school mathematics was beneficial to students' learning, both in terms of the conceptual benefits but also in relation to motivation and autonomy for learners

(Continues)

Table 3. (Continued)

Authors and year	Country	Participants and context	Aims	Design	Analysis	Main Findings
Kılıç (2015)	Turkey	Primary 5 ITE students	To explore pre-service primary teachers' ability to pose problems that can be solved by a specific problem-solving strategy	Exploratory study	Semantic analysis of posed problems and content analysis of interview transcripts	The majority of teachers were able to pose the appropriate problems. However, several characteristics of difficulties were displayed when teachers were not successful. These were posing problems which required an irrelevant strategy; being unable to find an answer to the posed problem and problems posed without the need for the specified problem-solving strategy
Kopparla and Capraro (2018)	USA	Primary 1 pupil	To explore the viability of using problem posing to understand the mathematical profile of pupils	Single case study	Descriptive analysis of pupils' work throughout the study	Problem-posing responses were helpful in identifying mathematical misunderstandings. Pupils were able to engage in more complex mathematics when posing contextualised problems in an area of interest. Problem posing may be used to evaluate students' misconceptions and also to explore mathematical understanding
Lavy and Bershadsky (2003)	Israel	Secondary	To explore the different kinds of problems posed by pre-service teachers using the 'what if not' strategy	Exploratory study	Descriptive analysis of written protocols; inductive analysis of clinical interviews and group discussion	When utilising the 'what if not' strategy, teachers changed the problem by changing one of the data components or changing the problem question. They had difficulty posing problems that were significantly different from the given problem
		28 ITE students				Teachers had to understand the mathematical content of a given problem in order to generate a new problem. The process of posing new problems highlighted conceptual misunderstandings

(Continues)

Table 3. (Continued)

Authors and year	Country	Participants and context	Aims	Design	Analysis	Main Findings
Lavy and Shriki (2010)	Israel	Secondary 25 ITE students	To explore changes in perceptions of ITE students regarding their mathematical knowledge during problem posing using ‘what if not?’	Exploratory study	Analytical induction of portfolios of learning using self-reflection	The process of posing new problems highlighted conceptual misunderstandings Students had difficulty in posing problems that were significantly different to example due to lack of conceptual knowledge Problem posing, and reflective writing developed teachers’ mathematical knowledge
Ozdemir and Sahal (2018)	Turkey	Primary 69 pupils	To explore the effect of teaching integers through the problem-posing approach on sixth-grade students’ academic achievement and mathematics attitudes	Explanatory design	Content analysis of observation	Problem posing was useful in revealing conceptual mistakes and errors of students. Although these hindered problem posing, the activity of posing problems could support conceptual understanding. Interaction through group work was helpful for supporting the development of mathematical knowledge
Ticha and Hospesova (2013)	Czech Republic	Primary 56 ITE students	Exploring problem posing as an educational and diagnostic tool	Exploratory study	Semantic analysis of posed problems. Descriptive analysis of students’ comments and coding of reflective statements in relation to problems posed	Problem posing is useful in motivating teachers to mathematical content. Problem posing highlighted poor mathematical content knowledge. The reflection process plays a significant role in supporting a deeper conceptual understanding

Table 4. Six step analysis guide (Braun & Clarke, 2006)

Step	Application to the current study
1. Familiarising yourself with your data	A process of reading and re-reading was undertaken in order to ensure full awareness of the papers
2. Generate initial categories	Initial categories were generated through an iterative and reflective process. The focus was on the results of the studies in relation to the review questions
3. Searching for themes	Four themes emerged from the data which had direct relevance to the aims of the study and coded data were divided accordingly
4. Reviewing themes	Again an iterative and reflective process was employed. This was done by reviewing the emergent themes in relation to the review questions and aims and discussion between the authors
5. Defining and naming themes	Naming themes were done in alignment with the review questions, quantitative analysis and aims of the study
6. Producing the report	Consideration was given as to how these data might be best presented alongside the quantitative data

repeated several times in order to ensure that the data were fully analysed through an iterative and reflective process as suggested by Nowell *et al.* (2017).

Quality appraisal

We conducted quality appraisal using the Critical Appraisal Skills Programme (CASP, 2018) qualitative checklist for the qualitative data (Appendix A), together with a modified version of this for the quantitative data (Appendix B). Whilst there is a CASP checklist available for quantitative randomised control studies, many of the categories relate to health interventions and were, therefore, not appropriate for the current study. Instead, we amended some categories and drew on the qualitative categories that were deemed suitable. This approach of combining qualitative and quantitative quality criteria is in line with recommendations by Caldwell *et al.* (2011).

Results

Quantitative data

Study characteristics. Fourteen studies involved pupils and six involved ITE students (primary or secondary mathematics). Publication dates ranged between 1997 and 2018. Studies were conducted in USA ($n = 6$), Turkey ($n = 5$), Kazakhstan ($n = 3$), Australia ($n = 2$), China ($n = 2$), Oman ($n = 1$), Indonesia ($n = 1$) and Iran ($n = 1$). The datasets contained 1,935 participants (1,517 pupils, 357 ITE students and 61 in-service teachers, respectively). All studies reported a problem-posing intervention and these ranged from 4 days to 2 years.

Outcome measures. A number of different measurements were used in the studies providing evidence for both cognitive (problem-solving performance, problem-posing

performance, mathematical achievement and mathematical creativity) and affective (attitude towards mathematics, beliefs, self-efficacy, interest & motivation and mathematical anxiety) benefits of problem posing to ITE students and pupils. These will be presented in relation to the research questions.

What are the benefits for learners of using mathematical problem posing in the curriculum? Cognitive benefits. Problem-solving performance improvements were identified in two studies for primary (Chen *et al.*, 2015; Kopparla *et al.*, 2018) and on for secondary pupils (Dickerson, 1999). Measurements used varied between previously published measures and author-developed measurements (see Table 2). Problem-posing improvements were highlighted in four studies for primary and two for secondary pupils (Xia *et al.*, 2008; Haghverdi & Gholami, 2015). Again, a range of measurement approaches was adopted.

Improvement in mathematical achievement was reported for pupils. For primary pupils, improvements were found in the areas of integers (Ozdemir & Sahal, 2018) and general mathematical knowledge (Chen *et al.*, 2015). Secondary school pupil improvements were found in the areas of probability (Demir, 2005), geometry (Haghverdi & Gholami, 2015; Mahendra *et al.*, 2017), general mathematical knowledge (Xia *et al.*, 2008; Kesan *et al.*, 2010; Guvercin *et al.*, 2014; Guvercin & Verbovskiy, 2014) and algebra (Walkington, 2017).

Affective benefits. Attitudinal improvements were identified for pupils. Primary pupils reported more positive attitudes to problem posing (Chen *et al.*, 2015) and problem solving (Priest, 2009; Chen *et al.*, 2015) and mathematics in general (Ozdemir & Sahal, 2018). Secondary school pupils reported more positive attitudes towards probability and also mathematics in general (Demir, 2005). Attitudes were measured using an assortment of instruments (see Table 2).

Primary pupils stated they were more motivated to engage (Priest, 2009) and had more interest in mathematics (Xia *et al.*, 2008). Secondary pupils reported increased interest in mathematics (Walkington, 2017) and improved motivation to engage in mathematics (Kesan *et al.*, 2010).

What are the benefits for ITE students using mathematical problem posing in the curriculum? Cognitive benefits. ITE students displayed performance improvements in both problem solving and problem posing. Mathematical achievement was also recorded in the areas of the level of integration and its applications (Akay & Boz, 2009) and fractions (Toluk-Uçar, 2009). One study detected improvements in mathematical creativity (Fetterly, 2010). The diversity of measurement instruments is reported in Table 2.

Affective benefits. ITE students reported more positive attitudes in general to mathematics (Akay & Boz, 2010) as well as lower levels of mathematics anxiety (Fetterly, 2010).

There was a positive change in beliefs for in-service teachers (Barlow & Gates, 2006) and ITE students about the teaching and learning of mathematics (Grundmeier, 2003; Fetterly, 2010) and about the relationship between problem posing and mathematics teaching (Grundmeier, 2003). ITE students also reported a positive change in self-efficacy in mathematics (Akay & Boz, 2010).

Overall these findings suggest a number of benefits for cognitive and affective aspects of learning mathematics both for pupils and teachers when problem posing is used. The broad range of measurement instruments, interventions and settings, which all converge to positive outcomes, provides evidence that problem posing is a potentially powerful tool in supporting the learning and teaching of mathematics. To strengthen further the interpretation of the evidence, effect size is shown where accessible or has been calculated by hand (within educational research, Hattie (2012) provided teachers with initiatives grounded on the analysis of effect sizes). The largest effect sizes for primary pupils were found for increased engagement, motivation and creativity. At the secondary level, there were large effect sizes for increased academic achievement. Whereas, for ITE students, the largest effect sizes were found for reducing mathematical anxiety, improving self-efficacy and improved conceptual understanding.

Qualitative data

Study characteristics. Three studies involved pupils and six involved ITE students (primary or secondary mathematics). Publication dates ranged between 1997 and 2018. Studies were conducted in the USA ($n = 3$), Israel ($n = 2$), Turkey ($n = 2$), Australia ($n = 1$) and Czech Republic ($n = 1$). The datasets contained 384 participants (287 ITE students and 97 pupils, respectively). It should be noted that 69 pupils in the study by Ozdemir and Sahal (2018) were also included in the quantitative data.

Identified themes. Four themes emerged from the qualitative synthesis: The interplay of mathematical knowledge and problem posing; the non-mathematical nature of problems; the importance of group work and reflection; and issues with the implementation of problem-posing tasks.

The interplay of mathematical knowledge and problem posing. Seven papers highlighted the interplay of mathematical knowledge and problem posing. Specifically, problem posing can help develop conceptual knowledge (English, 1997b; Ellerton, 2013) and can highlight conceptual misunderstanding (English, 1997b; Lavy & Shriki, 2010; Lavy & Bershadsky, 2003). However, mathematical knowledge is also a requirement of successful problem posing. Ozdemir and Sahal (2018, p. 125) provide evidence that problem posing can illuminate poor mathematical knowledge. Primary school pupils were asked to pose and solve integer problems. They were specifically asked to pose problems that were similar to those that they had solved during their lesson. The pupils were presented with a specific story or piece of information around which to base their problems. Posed problems, therefore, had to be mathematically solvable, relate to real-life contexts and be comprehensible. The following example met these criteria and also highlighted mathematical knowledge errors:

'Mrs Ayse parked her car in the car park area in the -3 floor. She goes to +5. How many floors does she go up?'

$$-3 - (+5) = -2$$

Ozdemir and Sahal (2018) reported that during this session, the mathematical error was identified by other pupils in the class. Similarly, Lavy and Shriki (2010) found that poor mathematical knowledge could impede the problem-posing activity of ITE students. Using a structured problem-posing approach (*what if not?*), students were given a problem and asked to develop a new problem through a specific process: (1) Solve the problem, (2) produce a list of attributes, (3) negate each attribute and suggest alternatives. When asked to pose new problems teachers tended to stick with trivial or simple forms. The problem-posing activity highlighted their lack of conceptual knowledge:

David: 'I decided to choose the alternative of a pentagon inscribed in a circle ... after examination of some drawings [using the software] I realized that I do not have the sufficient knowledge to prove it formally. So, I abandoned this course of inquiry and decided to focus on a square instead of a pentagon'. (p. 20)

ITE students in Grundmeier's (2015) study were able to reflect on the interplay of mathematical knowledge and problem posing for their future students after engaging in a problem-posing intervention:

By the problem-posing process, students begin to identify key terms and concepts that define a topic, and by structuring problems around these topics, they begin to make connections, which enhances the learning process. (p. 427)

Therefore, whilst problem posing might be impeded by a lack of mathematical knowledge, it can also be a tool to identify and address this, thus highlighting an important interplay between the two.

The non-mathematical nature of problems. Six papers addressed problems out-with the mathematical content that might impact on the problem-posing process, such issues being prevalent for both ITE students and school pupils. For example, lack of understanding of what a word problem is (Kopparla & Capraro, 2018; Ozdemir & Sahal, 2018) the inability to pose contextualised or sensible problems (Ticha & Hospesova, 2013; Kılıç, 2017), grammatical structure difficulties (Ellerton, 2013) and identifying the salient information in posing a problem (English, 1997b).

Kopparla and Capraro (2018) conducted an in-depth case study of the problem-posing development of one primary pupil during an intervention. They highlighted the lack of understanding of the nature of a mathematical problem by analysing tasks: 'There was a book fair on Wednesday and Saturday, which one sold books?' (p. 4). Similarly, Ozdemir and Sahal (2018, p. 125) noted the difficulties of constructing a word problem. When asked to pose an integer problem one pupil suggested the following: 'Erdem found the answer of the question that the teacher asked in mathematics exam wrong. Let's find the answer to this question'.

ITE students also display difficulties with translating the semantic structure of problems into meaningful text. This requires the students to be able to identify suitable surface-level (or contextual) information in which to embed the mathematical structure. For example, Kılıç (2017) asked participants to pose a word problem that could be solved using the find-a-pattern problem-solving strategy. One participant

who could not complete the task reported: 'In fact, I know the pattern-based problem-solving process and how to pose a problem. Of course I could produce many problems but I could not write any situations involving what you ask' (Kılıç, 2017, p. 783). Another participant posed a number problem and acknowledged: 'I think it is in relation to the question you asked but it is not a word problem' (p. 783). Other semantic-based problems could be mistaken for the lack of mathematical knowledge: 'Petr and Mirek are eating cakes that Granny has baked. Petr ate $\frac{3}{4}$ of a cake, Mirek $\frac{1}{2}$ more. How much did Mirek eat?' (Ticha & Hospesova, 2013, p. 139). The difficulty in this question is the placement of the word 'more' to indicate that Mirek ate $\frac{1}{2}$ more of the $\frac{1}{4}$ that was left after Petr ate $\frac{3}{4}$.

Problem posing can be used as a tool to highlight these difficulties and to address them. Although pupils might focus on the surface semantic structure of given problems, the process of learning of how to pose problems can highlight mathematical structure beyond surface content. For example, after a problem-posing intervention, English (1997b, p. 198) found a shift of focus from the surface content to structural content. When given problems and asked to categorise them at the beginning of the intervention it was clear that they were focussing on the surface content: 'They both start off, Bill has 52 marbles and Jan has 29 marbles, that's the starting sentence'. However, after the intervention, there was an acknowledgement that the surface information was not necessarily what made problems comparable: 'These two don't match—the similarity between them is that they are both talking about t-shirts'. Pupils could understand that just because two problems shared a similar topic it did not mean that they were similar in structure and should be grouped together.

These findings underline the significance of considering factors beyond the mathematical nature of problem posing that may hinder or support success. In particular, ITE students and pupils can have difficulty in understanding what a word problem is; appropriately contextualising information; and identifying what the salient information is within the problem.

The importance of group work and reflection. The importance of group work and reflection was illustrated in nine papers covering both ITE students (Lavy & Bershadsky, 2003; Lavy & Shriki, 2010; Ticha & Hospesova, 2013; Grundmeier, 2015; Kılıç, 2017) and pupils (English, 1997b; Kopparla & Capraro, 2018; Ozdemir & Sahal, 2018). This was evident through different processes. For example, group work allowed students to support one another's learning by highlighting errors and developing correct solutions to the problem. In Lavy and Bershadsky's (2003, p. 382) study, ITE students were asked to pose a problem using one alternative data component of a given problem. The following excerpt shows the development of student understanding of the original error through the process of discussion with peers and instructor:

Hina: *Instead of prism we can take parallelepiped or pyramid.*

Jacob: *Parallelepiped is a quadrangle-based prism so it cannot be suitable.*

Ran: *A pyramid is suitable, since all the other data components are consistent.*

Teacher: *Hina what do you think?*

Hina: *Ahaa [nodding with her head], parallelepiped is indeed a prism with quadrilateral base so it is not compatible with the rest of the data components.*

Pupils were also able to benefit from the collaborative input of peers during problem-posing sessions. For instance, when asked to pose a Cartesian product problem, one pupil wrote: 'Kelly has 4 pairs of shorts and 3 t-shirts. How many different pieces of clothing does she have?' to which her peer was able to *respond*: 'No, no, that's an add; that's not times' and provide an appropriate example: 'Kelly has 4 pairs of shorts and 3 t-shirts. How many different outfits can she make?' (English, 1997b, p. 207).

In studies where group work was not part of the design, the process of reflection through dialogue with the teachers or through journals supported the development of problem posing. In particular, when ITE students were asked to keep reflective journals of their problem-posing activities, they were able to consider the role and function of problem posing in their future careers: 'What I try to keep in mind most as I am problem posing is whether or not most students at a particular grade level will be able to find a solution with meaning and understanding' (Grundmeier, 2015, p. 426).

These findings provide support for pedagogical approaches that encourage collaborative interaction, either peer to peer or students to teachers. Even in the absence of peer interaction, the process of self-reflection can also support effective learning through problem posing.

Issues with the implementation of problem posing. All papers provided evidence that problem posing could be integrated into existing curricula, both for pupils and ITE students but implementation differed across studies. Approaches used included structured or semi-structured approaches such as 'what if not?' (e.g. Lavy & Bershadsky, 2003; Lavy & Shriki, 2010), contextualised free problem posing (e.g. Ticha & Hospesova, 2013; Kopparla & Capraro, 2018; Ozdemir & Sahal, 2018). Problem posing was also integrated through deconstructing components of given problems (e.g. English, 1997b) and through the use of specific problem-solving strategies (e.g. Kılıç, 2017).

However, specific issues with the implementation of problem solving to the curriculum were highlighted in seven studies, providing evidence that the type or structure of problem-posing activity can influence the extent to which ITE students and pupils were able to engage effectively. The type of problem-posing activity can have differential effects on successful engagement, with problem posing being enhanced for some (Kopparla & Capraro, 2018) but not for others (Kılıç, 2017). For example, Kopparla and Capraro (2018) highlighted the facilitative effect of free problem posing. In their study, pupils were able to solve more complex problems when they were situated in an area of personal interest, such as animals: 'So there were 12 pets in the pet store 3 people came and got two each how many are in the pet store now?' (p. 6). This is compared to the problem posing which involved drawing around a hand and posing relevant problems to which the pupils wrote: 'What is the length of my thumb [sic] finger in inches?' (p. 6). When asked to pose further problems, the pupils simply changed the finger. Related to this Ellerton (2013) found that whereas ITE students acknowledge the role of problem posing in mathematics, they preferred problem solving as they viewed this as an easier task: 'creating a problem can be difficult because it's hard to find numbers that will work out simple in the end' (p. 93).

The problem-posing structure was also reported as a factor in effective engagement. ITE students' learning problem posing within a structured environment (what if not?) found it difficult to pose problems that were very different, structurally and semantically to given problems (Lavy & Bershadsky, 2003; Ticha & Hospesova, 2013). However, using a structured then semi-structured approach was beneficial in supporting students to pose good problems:

'Working with the WIN strategy was like driving with a driving teacher. When your driving skills are poor and you do not have sufficient confidence and knowledge to drive by yourself, it is better to have a driving teacher sitting next to you. Since it was my first experience with inquiry tasks and problem posing in a computerized environment, the WIN strategy provided me with something to lean on—to work systematically and not to get lost within my endless trials, looking for interesting regularity'.
(Lavy & Shriki, 2010, p. 22)

These findings emphasise the importance of attending to the processes involved in the implementation of problem posing into the curriculum. They highlight that whilst problem posing per se may have benefits to both pupils and ITE students, particular attention should be afforded to the method in which it is delivered.

Methodological quality of studies

The highest score available for quality was 20. Quantitative scores ranged between 10 and 20 and qualitative studies ranged between 9 and 18, highlighting diverse methodological quality. The main area that was lacking in qualitative studies was the attention to ethical considerations and acknowledgement of the role of the researcher. Most qualitative studies were descriptive in nature and did not employ any systematic methodological analysis techniques.

Discussion

This study investigated the benefits of problem posing within the learning and teaching of mathematics. Findings highlighted strong effect sizes for increased motivation, creativity and engagement in primary pupils, increased achievement for secondary pupils and increased motivation, self-efficacy and conceptual understanding for ITE students. These will be discussed in relation to the research questions.

What are the benefits for learners of using mathematical problem posing in the curriculum?

Primary pupils. There is evidence from two studies (English, 1998; Priest, 2009) of a significant impact on attainment. English (1997b) found enhanced recognition of problem structures and diverse thinking. Specific problem-solving skills have also been enhanced (Chen *et al.*, 2015; Kopparla *et al.*, 2018). Chen *et al.* (2015) reported increased levels of interest and positive attitudes towards mathematics. Moreover, the study by Priest (2009) discovered that an intervention facilitated the re-engagement of disengaged pupils. This suggests that problem posing can make the learning of mathematics more enjoyable, thus helping to promote engagement. The impact of improvements in ability levels and mastery can also support engagement through increased self-efficacy (Bandura, 1977).

Secondary pupils. There is strong evidence from nine studies (Dickerson, 1999; Demir, 2005; Xia *et al.*, 2008; Kesan *et al.*, 2010; Guvercin *et al.*, 2014; Guvercin & Verbovskiy, 2014; Haghverdi & Gholami, 2015; Mahendra *et al.*, 2017; Walkington, 2017) of a significant impact on attainment. For example, Demir (2005) found that participants performed significantly better on a probability test. Similarly, Walkington (2017) reported significant improvement in algebraic performances. These studies suggest that problem posing can enhance conceptual understanding and therefore raise attainment. Furthermore, there is strong evidence from four studies (Demir, 2005; Xia *et al.*, 2008; Guvercin *et al.*, 2014; Guvercin & Verbovskiy, 2014; Chen *et al.*, 2015) of improved levels of interest and positive attitudes towards mathematics. Likewise, there is reasonable evidence from two studies (Kesan *et al.*, 2010; Guvercin & Verbovskiy, 2014) of increased levels of motivation, cognition and flexible thinking which infer that problem posing can develop more positive attitudes towards mathematics as well as enriching pupils' thinking. Notably, Dickerson (1999) found strong evidence of improved problem-solving ability.

What are the benefits for ITE students using mathematics problem posing?

There is strong evidence from three studies (Abu-Elwan, 2002; Akay & Boz, 2009; Toluk-Uçar, 2009) of a significant impact of achievements such as improved problem-solving performance and conceptual knowledge. Toluk-Uçar (2009) found that problem posing can result in a change in beliefs on the nature of mathematics. Similarly, there is reasonable evidence from two studies (Akay & Boz, 2010; Fetterly, 2010) that problem posing can enhance creativity and self-efficacy, foster positive beliefs and reduce anxiety. Collectively, these studies suggest that students' primary teachers in particular can benefit from problem posing. Theorists propose a critical role for self-efficacy and beliefs in relation to human performance. Bandura (1977, p. 3) refers to 'self-efficacy' as 'beliefs in one's capabilities to organise and execute the courses of action required to produce given attainments'. Problem posing has the promise of nurturing teachers' thinking and their future professional practice.

To what extent should problem posing be embedded within the mathematical framework of CfE?

The aim of the integrative review was to consider qualitative evidence that can provide some explanation as to the mechanisms that might account for the quantitative findings. In doing so, it is possible to provide some level of guidance as to the type of issues that curriculum architects should take cognisance of when designing a formal implementation of problem posing.

Quantitative findings suggested that problem-posing interventions can improve both problem-posing and problem-solving skills. Qualitative findings indicate that this might be due to the interplay of skills required for the two processes. Mathematical knowledge is required for successful problem solving and it is the process of problem posing that can highlight difficulties in this. Grundmeier (2015) notes that it is through the integration of problem posing with problem solving that ITE students

can develop their conceptual knowledge. Within the classroom, Ozdemir and Sahal (2018) found that pupils can also benefit since problem posing can be used as a tool to uncover their conceptual misunderstandings. This supports previous research findings that problem-posing and problem-solving skills are highly correlated (Silver & Cai, 1996), more able problem solvers pose more complex problems (Ellerton, 1986) and more able pupils remember structural rather than surface information on solved problems (Krutetskii, 1976). Therefore, when considering the inclusion of problem posing within CfE it would be appropriate to teach and integrate the skill with problem solving, taking account of differential student ability in problem solving and its potential impact on problem-posing ability.

Yet, the current curricula structure does not possess the intended flexibility to sustain the coalescing of new research perspectives. Although it may be perceived that it is straightforward to initiate a change in professional practice, it is another matter to navigate the trajectory of a transformational change in educational policy. One method to achieve this is to combine both a descriptive and prescriptive approach to the mathematics framework that will ensure conceptualisation and operationalisation of problem posing. If problem-posing activities are to play a more central role in classrooms, they should permeate the entire curriculum (Bonotto & Del Santo, 2015; Cai *et al.*, 2015).

The account should also be taken of other difficulties that ITE students and pupils might face when engaging in problem-posing activities. The semantic nature of problems was highlighted as a difficulty for both ITE students and pupils, suggesting that some scaffolding is required in the process initially (e.g. Ticha & Hospesova, 2013; Kılıç, 2015). However, the process of continued engagement with problem-posing activities, alongside problem solving helps learners to uncover structural aspects of problems, rather than focussing on surface information (English, 1997b). Indeed, problem posing has been used effectively as a formative assessment tool for understanding students' learning (e.g. Kotsopoulos & Cordy, 2009). Helping teachers gain valuable insights into pupils' mathematical thinking that can only serve to positively impact on future classroom experiences.

Expanding the benchmark for mathematics is, therefore, a necessary step to provide details on the standards expected for each level. Professional dialogue may act as a springboard for practitioners to innovate and enhance their practice since engaging with research is a requirement for full registration. Problem posing can help underpin a whole school approach to effective monitoring and tracking of skills detailed above.

The pedagogical approach should also be considered with any implementation. Affect is an important factor in mathematics learning. The quantitative analysis suggested that areas of affect such as self-efficacy, motivation and anxiety in mathematics might be supported through problem posing. Many of the qualitative studies utilised collaborative approaches and those which did not use reflective diaries (e.g. Lavy & Shriki, 2010; Grundmeier, 2015; Kopparla & Capraro, 2018). The process of interacting with others gives learners access to multiple cognitive processes, as well as developing important social interaction skills. Theoretical support for the importance of group work in learning comes from socio-cognitive theories whereby peers are able to scaffold learning (e.g. Vygotsky, 1978; Slavin, 1989). The mastery that

develops from this type of successful interaction can then translate to higher levels of self-efficacy (Bandura, 1977).

Mastery and higher self-efficacy can also promote positive attitudes. Learners display heterogeneous attitudes towards the efficacy of learning through structured approaches. Structured approaches might initially support ITE students in their learning (Lavy & Shriki, 2010). However, pupils might find free problem-posing approaches more interesting due to the 'real life' nature of them (Kopparla & Capraro, 2018). Currently, there is no defined problem-posing model that describes the *process* of problem posing (Cai, *et al.*, 2015). It is therefore important that educators are able to understand the differences between types of problem-posing activities, as well as being cognisant of the relevance of these to individual groups of learners.

The findings presented here suggest that problem posing is a powerful pedagogy for raising attainment and achievement in mathematics. In order to be at the heart of learning and teaching, problem posing should be accredited equal status to problem solving. In Scotland, teachers draw upon national benchmarks (Education Scotland, 2017) which outline the knowledge, understanding and key skills for each pathway.

However, from the professional experience of the first author, there appears to be disparity between the holistic values and principles advocated by the research community and those implemented in Scottish classrooms. For instance, there is an overplaying of examination techniques, which consequently, have suppressed the cultivation of creativity. The Scottish Government has a responsibility to recalibrate how they measure mathematical success in schools. The curriculum should be centred on rich tasks that encourage higher levels of thinking and reasoning as opposed to the saturation of routine procedural or computational activities. Mathematical problem posing yields such tasks.

Limitations and future research

Whilst systematic integrative reviews have inherent strengths such as the implementation of a comprehensive search strategy and strict eligibility criteria, limitations still exist. This review was restricted by the search terms and the period of published research. It was also limited to English language texts only. Furthermore, publication bias may have influenced included studies. We addressed this in part by searching grey literature such as unpublished theses. However, further relevant articles may have been missed.

Future studies should assess the impact of mathematical problem posing within the spectrum of Scottish education, including ITE students at both primary and secondary level.

Conclusions

Through the use of an integrative review of qualitative and quantitative research, we examined the legitimacy of infusing problem posing within the national curricula of Scotland to improve the learning and teaching of mathematics. The results are relevant to practitioners and policy makers. Findings suggest that there is compelling

evidence to support the introduction of problem posing within the framework of CfE, consistent with previous research (e.g. Stoyanova, 2003; Bonotto, 2013; Leung, 2013, 2016; Singer *et al.*, 2013; Cai *et al.*, 2015). Problem posing should be compartmentalised as a unique cognitive activity since it requires learners to extend their mathematical thinking beyond problem-solving procedures (Cai & Hwang, 2002) as well as resonating with a social constructivist paradigm. However, consideration should be given to findings from qualitative studies when designing implementation.

The key to pupils being able to construct worthwhile mathematical problems relies on the professional actions of teachers in enacting tasks, reflections upon practice and inventing innovative steps in instruction (Leung, 2016). The interaction between problem solving and problem posing should be made explicit; thereby engendering a much welcomed by-product that more teachers will be able to distinguish the profile of a mathematical problem. In pragmatic terms, underpinning such an outcome is an obligation for teachers to acquire new conceptual understandings and pedagogical knowledge (English, *in press*). Indeed, it is essential that problem posing is perceived as a mechanism to nurture creativity, independence and originality. Manifesting such an ideology requires a shift in 'political' accountability so that mathematics is taught for meaning rather than for measurement (Maclellan, 2014). A major policy priority should, therefore, be to plan for assimilating problem posing within mathematics education in Scotland. Although a number of changes need to be made, primary and secondary teachers should not be expected to function autonomously without the creation of high-quality professional learning opportunities.

Conflict of Interest

No potential conflict of interest is reported by both authors.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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Appendix A. Quality criteria for the quantitative studies

Paper	Was there a clear statement of the aims of the research?	Is a quantitative methodology appropriate?	Was the research design appropriate to address the aims of the research?	Was the recruitment strategy appropriate to the aims of the research?	Was there a control group?	Were participants assigned randomly to control/experimental?	Have ethical issues been taken into consideration?	Was the data analysis sufficiently rigorous?	Are the results generalisable to the study population?	Is there a clear statement of findings?	Total Score
Abu-Elwan (2002)	2	2	2	2	2	1	1	2	2	2	18
Akay and Boz (2009)	2	2	2	2	2	2	1	2	2	2	19
Akay and Boz (2010)	2	2	2	2	2	0	2	2	2	2	18
Barlow and Gates (2006)	2	2	2	2	0	0	1	2	2	2	15
Chen <i>et al.</i> (2015)	2	2	2	2	2	0	2	2	1	2	17
Demir (2005)	2	2	2	2	2	2	2	2	2	2	20
Dickerson (1999)	2	2	2	1	2	2	2	1	2	2	18
English (1998)	2	2	2	2	2	2	0	2	2	2	18
Fetterly (2010)	2	2	2	2	1	2	2	2	2	2	19
Grundmeier (2003)	2	2	2	2	0	0	2	0	2	2	14
Guvercin <i>et al.</i> (2014)	2	2	2	2	2	0	0	2	2	2	16
Guvercin and Verbovsky (2014)	2	2	2	2	2	0	0	2	0	2	14
Haghverdi and Gholami (2015)	2	2	2	2	0	0	0	1	0	1	10
Kesan <i>et al.</i> (2010)	2	2	2	2	2	0	0	2	0	2	14
Kopparla <i>et al.</i> (2018)	2	2	2	2	0	2	2	2	2	2	18
Mehendra <i>et al.</i> (2017)	2	2	2	2	0	2	0	2	1	1	14
Ozdemir and Sahal (2018)	2	2	2	2	2	2	0	0	2	2	16
Priest (2009)	2	2	2	2	2	2	2	1	2	2	19
Toluk-Uçar (2009)	2	2	2	2	2	1	1	2	1	2	17
Walkington (2017)	2	2	2	1	2	1	0	2	2	2	16
Xia <i>et al.</i> (2008)	2	2	2	2	2	0	0	2	2	2	16

Code: 0 = No, 1 = Not sure, 2 = Yes

Appendix B. Quality criteria for the qualitative studies

Paper	Was there a clear state-ment of the aims of the research?	Is the meth-odology appropri-ate?	Was the re-search design appropriate to address the aims of the research?	Was the recruitment strategy ap-propriate to the aims of the research?	Was the data collected in a way that addressed the research issue?	Has the relation-ship between researcher and par-ticipants been adequately considered?	Have ethical is-sues been taken into consid-eration?	Was the data analysis sufficiently rigorous?	Is there a clear state-ment of findings?	Is the research valuable?	Total score
Ellerton (2013)	2	2	2	1	2	0	1	0	2	2	14
English (1997b)	2	2	2	2	2	0	1	1	2	2	16
Grundmeier (2015)	0	1	1	1	1	0	1	1	2	1	9
Kiliç, (2017)	2	2	2	2	2	0	1	2	2	2	17
Kopparla and Capraro (2018)	2	2	2	2	2	0	2	2	2	2	18
Lavy and Bershadsky (2003)	2	2	2	2	2	0	0	1	2	2	15
Lavy and Shriki (2010)	2	2	2	2	2	0	0	2	2	2	16
Ozdemir and Sahal (2018)	2	2	2	2	2	0	0	2	2	2	16
Ticha and Hospesova (2013)	1	2	1	1	1	0	0	1	1	2	10
Code: 0 = No, 1 = Not sure, 2 = Yes											